Pair creation by photons in the field of an electron or positron pulse of high density

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The spectrum of electron-positron pairs created by photons colliding with an electron or positron pulse of a high density is calculated with a quasiclassical method. Using the crossing symmetry of the pair-creation process and beamstrahlung, we derive the formulas applicable in the classical and the quantum domain of parameters. Scaling laws for the total probability and for the spectrum of the created particles are derived. Our numerical results are in good agreement with those of a previous, purely quantum-mechanical calculation.

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I. INTRODUCTION

Dense pulses of electrons and positrons, consisting of about 10^{10} particles, are created in modern colliders. When these pulses pass through one another a considerable portion of their kinetic energy is radiated as photons. This radiative process was called beamstrahlung [1, 2]. The interest in beamstrahlung has been greatly increased by prospects of building high-energy linear colliders for electron and positron beams in the TeV region. The quantum-mechanical calculation performed in [1, 2] predicts approximately 20% fractional energy loss for colliders planned for the nearest future. We confirmed this result in our recent work, using a quasiclassical method [3].

This quasiclassical method is based on the assumption that the scattering of the electron or the positron in collision with the dense pulse of the opposite charge proceeds quasiclassically in the mean field created by all particles of the pulse. The fractional energy loss and the shape of the beamstrahlung spectrum are substantially different from the ordinary bremsstrahlung spectrum. This difference originates due to the fact that the bremsstrahlung coherence length of the projectile particle substantially surpasses the interparticle distances in the pulse. This fact permits one to consider the projectile-particle scattering and the corresponding radiation as a collective phenomenon taking place in the mean field of the pulse.

In the present work we discuss the pair-creation process in the collision of a high-energy photon with a charged pulse of electrons or positrons. This problem is closely related to the beamstrahlung problem, because the pair creation by photon and the beamstrahlung are the two crossing processes. The amplitudes of these processes can be obtained from one another simply by changing the initial and the final states of the particles involved in the collision. Indeed, by interchanging the photon and the initial electron states in the beamstrahlung amplitude, we derive the amplitude for pair creation. The crossing symmetry provides us with a simple way to solve this problem. We developed the quasiclassical approximation for the beamstrahlung problem in [3]. Now using the crossing symmetry, we transform the beamstrahlung formulas to the initial and final states of the particles, which are appropriate for pair creation.

The discussed problem is interesting as a fundamental process of quantum electrodynamics. It might be observed experimentally in planned linear electron-positron colliders of TeV energy. Colliding high-energy particles radiate predominantly high-energy photons as the beamstrahlung spectrum possesses a sharp maximum near the edge [1, 3]. Such a high-energy photon can create with a certain probability an electron-positron pair in the field of the pulse. We demonstrate in our work that this probability is large. It can approach unity over the length of the pulse. We prove this result, considering a collision of a projectile high-energy photon with a pulse. Parameters of the pulses are presented in [1–4] for some existing and planned colliders.

In our work we have obtained the energy distribution of the created electrons or positrons for different photon energies. The shape of the spectrum changes drastically with increasing photon energy. The origin of this change is that with increasing energy we leave the classical limit and approach the quantum limit.

The total probability of pair creation as well as the spectrum are scaling functions. The scaling parameter of the process is a complicated combination of values, characterizing the pulse and the collision. The scaling laws for the total probability and for the spectrum are obtained and analyzed in our paper. We performed the numerical calculation of the spectrum and the total probability as a function of the scaling parameter. The obtained numerical results are in good agreement with those of the purely quantum-mechanical treatment of the process [5, 6]. This agreement transparently shows the quasiclassical nature of the considered pair-creation process.

II. ENERGY DISTRIBUTION

We wish to perform the calculation of the energy distribution of positrons or electrons, using the crossing symmetry between pair creation and beamstrahlung. The amplitude for pair creation by a photon can be obtained from the amplitude of the beamstrahlung by performing the following substitution:

$$\varepsilon; \mathbf{p} \longrightarrow -\varepsilon_+; -\mathbf{p}_+,$$

$$\varepsilon'; \mathbf{p}' \longrightarrow \varepsilon_-; \mathbf{p}_-, \tag{1}$$

$$\omega; \mathbf{k} \longrightarrow -\omega; -\mathbf{k} .$$

Here $(\varepsilon; \mathbf{p})$, $(\varepsilon'; \mathbf{p}')$ are the energy and the momentum of the electron in the initial and in the final states; $(\omega, \mathbf{k})$, $(\varepsilon_-; \mathbf{p}_-)$, $(\varepsilon_+; \mathbf{p}_+)$ are the same for the photon and the created electron-positron pair, respectively.

The expression for the probability of a quantum-mechanical process contains the square of the amplitude and the phase-space factor of particles in the final state [7]. The phase-space factors are different in the beamstrahlung and in the pair-creation process due to the different sets of particles in the final states. The ratio of the phase-space factors in the two processes is equal to

$$\frac{d^3k}{d^3p_+} = \frac{\omega^2}{p_+^2 c^2}\frac{d\omega}{d\varepsilon_+}. \tag{2}$$

The ultrarelativistic dispersion relation ($\varepsilon_+ = p_+/c$) for the positron energy has been used in (2). We use atomic units ($\hbar = m_e = |e| = 1$).

Let us now transform the probability for beamstrahlung into the probability of pair creation. It was demonstrated in [3] that at each instant the mean electric field of the pulse in its rest frame is equivalent to the effective uniform magnetic field, acting on the projectile particle. The probability of the radiation in the uniform magnetic field was calculated by the quasiclassical method [7, 8]. This result reads

$$\frac{dP_\gamma}{d\omega} = -\frac{e^2 m^2 c^3}{\varepsilon^2}\left\{ \int_\xi^\infty Ai(z)\,dz + \left(\frac{2}{\xi} + \frac{\omega}{\varepsilon}\chi\,\xi^{1/2}\right)\mathrm{Ai}'(x)\right\}, \tag{3}$$

$$\varepsilon = \varepsilon' + \omega, \quad \xi = \left(\frac{\omega}{\varepsilon'\chi}\right)^{2/3}, \quad \chi = \frac{\omega_0}{\varepsilon}\left(\frac{\varepsilon}{mc^2}\right)^3 .$$

The parameter ω_0 is defined as

$$\omega_0 = \frac{c|e|H}{\varepsilon},$$

where H is the strength of the uniform magnetic field acting on the particle of mass m and charge e.

The Airy function $\mathrm{Ai}(x)$ is depicted as

$$\mathrm{Ai}(x) = \frac{1}{\sqrt{\pi}}\int_0^\infty \cos(ux + u^3/3)du .$$

Using the relations (1) and the ratio of the phase-space factors (2), we derive from (3) the probability of pair creation in the uniform magnetic field

$$\frac{dP_p}{d\varepsilon_+} = \frac{e^2 m^2 c^3}{\omega^2}\left\{\int_{\xi_p}^\infty \mathrm{Ai}(z)\,dz + \left(\frac{2}{\xi_p} - \chi_p\,\xi_p^{1/2}\right)Ai'(\xi_p)\right\}, \tag{4}$$

$$\omega = \varepsilon_+ + \varepsilon_- , \quad \xi_p = \left(\frac{m^3c^5\omega}{|e|H\varepsilon_+\varepsilon_-}\right)^{2/3} , \quad \chi_p = \frac{|e|H\omega}{m^3c^5}.$$

Let us now apply this result to the pair-creation process in the mean field of a pulse. This field is supposed to have an axial symmetry. The strength of the electric field of the pulse in its rest frame is equal to [3]

$$\mathbf{E} = \frac{2Ne}{LR^2}\mathbf{r} , \tag{5}$$

where $\mathbf{r}$ is the radius vector in the transverse plane of the pulse, R and L are the pulse radius and length, and N is the number of particles. It was shown in [3] that the electric field of the pulse influences the trajectory of the high-energy projectile only weakly. Therefore, we assume the particle trajectories to be straight lines. This fact permits us to obtain the total averaged probability of pair creation by integrating the local probability (4) over the linear trajectory of the photon in the pulse and averaging this expression over the impact parameter.

Let us carry out this calculation in the laboratory frame. Integrating (4) over time, we derive the probability for pair creation on a single photon trajectory. This integration is trivial. It adds an additional factor $L\gamma^{-1}/2c$ to the expression (4). The averaging of the probability over the impact parameters gives the result

$$\left\langle\frac{dP_p}{d\varepsilon_+}\right\rangle = \frac{e^2}{c}\frac{L\gamma^{-1}}{c}\frac{(mc^2)^2}{\omega^2} \times \int_0^1 dy\, y\left\{\int_{\xi_p}^\infty \mathrm{Ai}(z)\,dz + \left(\frac{2}{\xi_p} - \chi_p\,\xi_p^{1/2}\right) \times \mathrm{Ai}'(\xi_p)\right\} . \tag{6}$$

Here the parameters are introduced as follows:

$$\chi_p = \frac{|e|E'\omega}{m^3c^5}, \quad \xi_p = \left(\frac{\eta}{\chi}\right)^{2/3} , \quad \eta_p = -\frac{\omega^2}{\varepsilon_-\varepsilon_+} ,$$

$$E' = 2\gamma E, \quad E = \frac{2Ney}{RL}, \quad y = \frac{r}{R} .$$

The electric field E, acting on the photon in the transversal plane of the pulse, is taken from (5), and E' is the strength of the field in the laboratory frame. We suppose that E' is equivalent at each instant to the effective uniform magnetic field. We have discussed this point in detail in [3].

In [1, 3] scaling laws for beamstrahlung have been proposed. Similar laws exist in the case of pair creation as well. Introducing the energy of the incident photon instead of the energy of the projectile particle in the beamstrahlung scaling parameter C_γ [3], we derive the pair-creation scaling parameter C_p in the form

$$C_p = \frac{m^3 c^5 y}{|e| E(2\gamma\omega)}. \tag{7}$$

The fractional variable x is defined in the case of pair creation as

$$x = \frac{\varepsilon_+}{\omega} ,$$

$$1 - x = \frac{\varepsilon_-}{\omega} .$$

Expressing ξ_p and χ_p via x, C_p and y, we come to the result

$$\xi_p = \left(\frac{C_p}{x(1-x)y}\right)^{2/3} , \qquad \chi_p = \frac{y}{C_p} . \tag{8}$$

Then, introducing the fine-structure constant $\alpha = |e|^2/c$ and the constant $\beta = N|e|^2/Rmc^2$, we represent the probability (6) in the form

$$\left\langle \frac{dP_p}{dx_p} \right\rangle = \alpha\beta C_p R_p(C_p; x_p) ,$$

$$R_p(C_p; x_p) = 4 \int_0^1 dy\, y \left\{ \int_{\xi_p}^{\infty} \mathrm{Ai}(z)\, dz + \left(\frac{2}{\xi_p} - \frac{y}{C_p} \xi_p^{1/2} \right) \times \mathrm{Ai}'(\xi_p) \right\}. \tag{9}$$

Let us simplify the double integral in $R_p(C_p; x_p)$. Using the new integration variable ξ_p instead of y, we derive

$$R_p(C_p; x_p) = R_1(U_p) - \frac{(x_p - 1)^2 + x_p^2}{x_p(1 - x_p)} R_2(U_p) . \tag{10}$$

Here $R_1(U_p)$, $R_2(U_p)$ are equal to

$$R_1(U_p) = 6U_p^3 \int_{U_p}^{\infty} d\xi_p \frac{1}{\xi_p^4} \int_{\xi_p}^{\infty} dz\, \mathrm{Ai}(z) ,$$

$$R_2(U_p) = 6U_p^3 \int_{U_p}^{\infty} d\xi_p \frac{1}{\xi_p^5} \mathrm{Ai}'(\xi_p) ,$$

$$U_p = \frac{C_p^{2/3}}{x_p^{2/3}(1 - x_p)^{2/3}} .$$

The double integral in $R_1(U_p)$ can be reduced to a single integral. Changing the order of integration in $R_1(U_p)$, one comes to the result

$$R_1(U_p) = 2 \int_{U_p}^{\infty} dz\, \mathrm{Ai}(z) \left\{ 1 - \frac{U_p^3}{z^3} \right\} . \tag{11}$$

Let us compare the obtained results with those derived for beamstrahlung in [3]

$$\left\langle \frac{dP_\gamma}{dx_b} \right\rangle = \alpha\beta C_b R_b(C_b; x_b) ,$$

$$R_b(C_b; x_b) = -R_1(U_b) - \frac{1 + x_b^2}{x_b} R_2(U_b) . \tag{12}$$

Here the constants α and β and the functions $R_1(U_b)$, $R_2(U_b)$ are the same as in (9); the scaling parameter C_b and the fractional variable x_b are defined as

$$C_b = \frac{m^3 c^5 RL}{4N|e|^2 \gamma\varepsilon} ,$$

$$x_b = 1 - \frac{\omega}{\varepsilon} ,$$

and the variable U_b is equal to

$$U_b = C_b^{2/3} \left(\frac{1 - x_b}{x_b} \right)^{2/3} .$$

The results (9) and (12) have the same structure. This might be expected due to the crossing symmetry between the two processes. But nevertheless the scaling functions $R_b(C_b; x_b)$ and $R_p(C_p; x_p)$ show a different behavior as functions of scaling parameters C_b and C_p and the fractional variables x_b; x_p. We discuss their behavior in more detail below, where we present numerical results.

III. TOTAL PROBABILITY

We consider in this section the total probability of pair creation by a photon in a pulse field. This probability is a function of the scaling parameter C_p. It can be obtained by integrating the energy distribution (9) over the variable $x_p = \varepsilon_+/\omega$,

$$P = \alpha\beta C_p \int_0^1 dx_p R_p(C_p; x_p). \tag{13}$$

Let us define the integral in (13) as

$$I = \int_0^1 dx_p R_p(C_p; x_p) = I_1 + I_2, \tag{14}$$

where

$$I_1 = \int_0^1 dx_p R_1(U_p) ,$$

$$I_2 = -\int_0^1 dx_p \frac{(x_p - 1)^2 + x_p^2}{x_p(1 - x_p)} R_2(U_p) .$$

It is sufficient to carry out the integration in I_1, I_2 only over half of the integration interval due to the symmetry of the integrands relative to the point 0.5. Taking U_p as a new variable instead of x_p and substituting $R_1(U_p)$ from (11), we come to the following result:

$$I_1 = 6C_p \int_{(4C_p)^{2/3}}^{\infty} dU_p \frac{U_p^{1/2}}{U_p^3 \sqrt{1 - 4C_p/U_p^{3/2}}} \times \int_{U_p}^{\infty} dz\, \mathrm{Ai}(z) \left\{ 1 - \frac{U_p^3}{Z^3} \right\} . \tag{15}$$

Changing the integration order in (15) and integrating over U_p, we obtain

$$I_1 = 2\int_{(4C_p)^{2/3}}^{\infty} dz\,\mathrm{Ai}(z)\left\{\frac{1}{z^{3/4}\sqrt{z^{3/2}-4C_p}}\left(1-\frac{2C_p}{z^{3/2}}\right)-\frac{8C_p^2}{z^3}\ln\left(\frac{z^{3/4}+\sqrt{z^{3/2}-4C_p}}{(4C_p)^{1/2}}\right)\right\} . \tag{16}$$

The integral I_2 can be simplified in a similar manner. After introducing the new variable U_p, we obtain the double integral

$$I_2 = -18\int_{(4C_p)^{2/3}}^{\infty} du_p \frac{U_p^2}{\sqrt{1-4C_p/U_p^{3/2}}}\left(1-\frac{2C_p}{U_p^{3/2}}\right)\int_{U_p}^{\infty} dz\,\frac{\mathrm{Ai}'(z)}{Z^5} . \tag{17}$$

Changing the order of integration in (17) and integrating over U_p, we derive

$$I_2 = -6\int_{(4C_p)^{2/3}}^{\infty} dz\,\frac{\mathrm{Ai}'(z)}{z^5}\left\{z^{3/4}\sqrt{z^{3/2}-4C_p}(z^{3/2}+4C_p)+8C_p^2\ln\left(\frac{z^{3/4}+\sqrt{z^{3/2}-4C_p}}{(4C_p)^{1/2}}\right)\right\} . \tag{18}$$

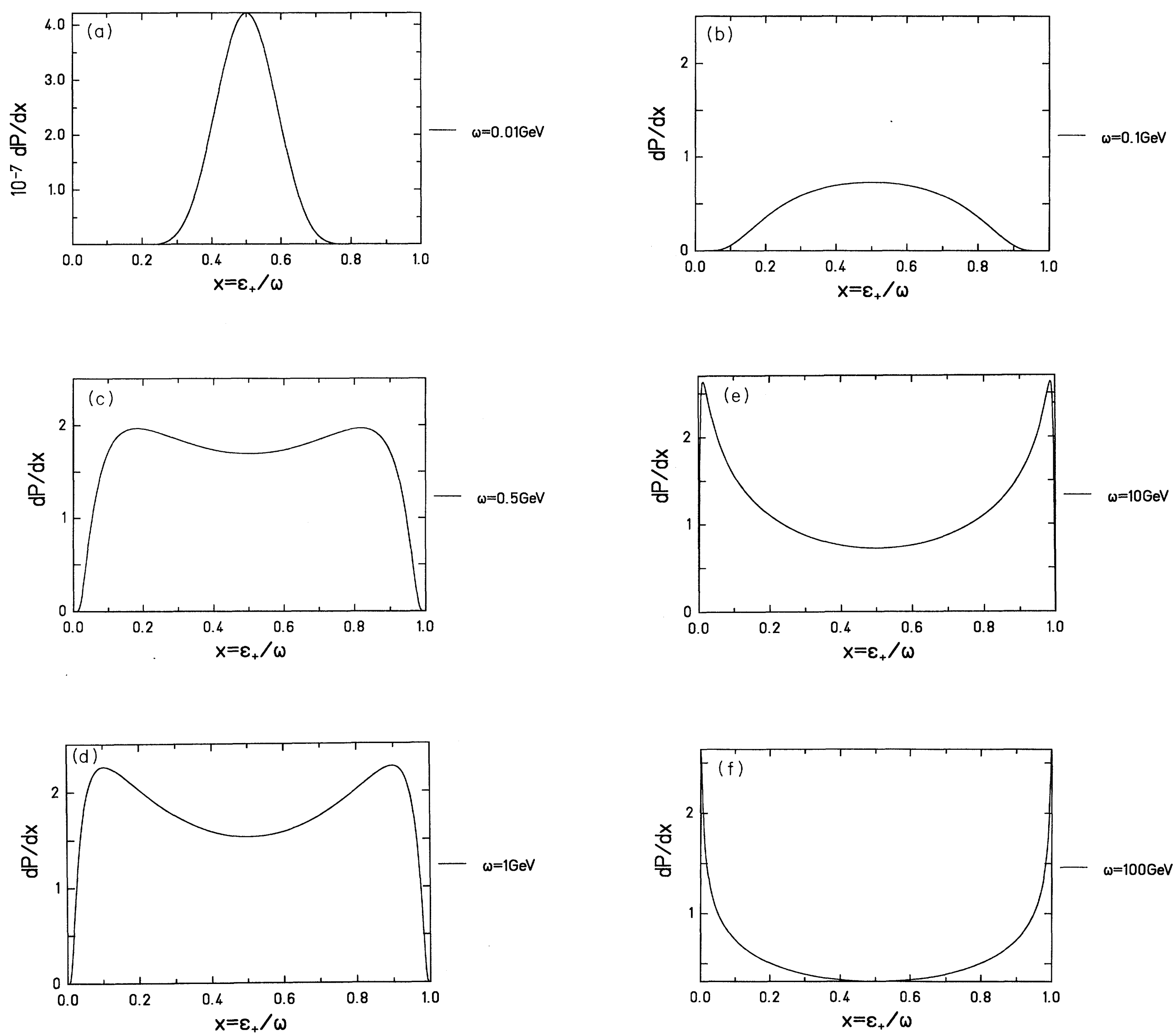

FIG. 1. Positron-energy distributions for different energies of the incident photon: (a) 0.01 GeV, (b) 0.1 GeV, (c) 0.5 GeV, (d) 1 GeV, (e) 10 GeV, and (f) 100 GeV.

The expressions (13) and (14) together with (16) and (18) describe the total probability of pair creation as a function of the scaling parameter C_p.

IV. NUMERICAL RESULTS

In this section we present the results of our numerical calculations for the energy distribution and the total probability of pair creation. The parameters characterizing the pulse enter the constant β and the scaling parameter C_p. The scaling parameter is also influenced by the energy of the incident photon. Therefore, varying the energy of the incident photon, we obtain different C_p and as a result different energy distributions of positrons. Large C_p or small photon frequencies correspond to the classical region, where the probability of pair creation is exponentially small. Small C_p or large photon frequencies characterize the domain, where a quantum-mechanical treatment of the process is necessary. The shape of the energy distribution of the positrons is different in the classical and in the quantum domain of parameters. The energy distribution of positrons for different photon energies is shown in Figs. 1(a)–1(f). All the curves are symmetric with respect to the point $x = 0.5$. This is clear, as the probability for the electron and the positron to carry a given fraction of energy x is equal. This implies that the probabilities of observing the positron with energy fraction x and $1 - x$ are equal.

The broad maximum appears in the middle of the positron energy distribution in the classical limit [see Figs. 1(a) and 1(b)]. In the quantum domain the distribution possesses the two symmetrical maxima near the edges [see Figs. 1(e) and 1(f)]. These features of the energy distribution are closely connected with those appearing in beamstrahlung spectra. Beamstrahlung is dominated by the small fractional energy loss in the classical case. That means that the electrons in the initial and in the final state have approximately the same energy. Therefore, it is rather natural to expect a small energy difference between the electron and the positron in the crossed process of pair creation, when its parameter C_p is in the classical domain. In contrast, the beamstrahlung shows large electron energy losses in the quantum domain. The energy of the electron in the initial state is substantially larger than in the final state. The corresponding large energy difference between the electron and the positron reappears in the pair-creation spectra. This is seen in Figs. 1(e) and 1(f), where sharp maxima exist near the edges of the energy distribution for positrons.

We have also calculated the total probability of pair creation, using (13), (14), (16) and (18). The results of our calculations are shown in Fig. 2. In the classical domain the probability decreases exponentially with an increase of the scaling parameter C_p, while in the quantum domain (small C_p) we have a power decay of the probability. In Fig. 2 we compare the results of our calculations with those having been derived purely quantum mechanically [5]. Our consideration shows that the quasiclassical treatment of pair creation by photons in the pulse field is a simple and reliable method able to describe correctly the physical nature of the process.

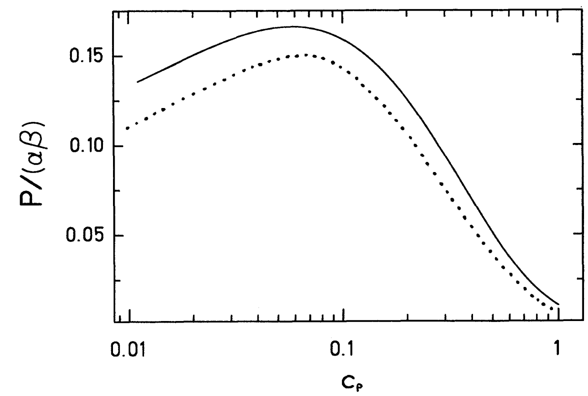

FIG. 2. Total pair-creation probability as a function of the scaling parameter C_p. The solid curve is the result of our calculation. The dotted curve is taken from [1].

V. CONCLUSION

In our paper we have discussed pair creation by photons in collision with the high-energy electron or positron pulse. This process is interesting to consider since high-energy pulses with large charge density will be created in the next generation of colliders. We have calculated the energy spectrum of positrons in the classical and in the quantum domain of parameters, demonstrating that the spectra are substantially different in these regions. We provide the simple qualitative explanation of this difference based on the crossing symmetry between pair creation and beamstrahlung. We have also calculated the energy distribution of positrons in the intermediate region of the scaling parameters, demonstrating the transformation of the energy distribution between the classical and the quantum domains. We derived the total probability for pair creation and demonstrated how it changes with the variation of the scaling parameter C_p. Our results are in good agreement with those derived by the purely quantum-mechanical method.

We have considered only the creation of electron-positron pairs. Our method may be easily extended to describe the creation of other lepton pairs, like μ^+-μ^-. The probability of these processes is smaller due to the higher masses of the created particles. One can also imagine more complicated electromagnetic processes in the fields of high-energy electron-positron pulses. The investigation of these processes poses an interesting and important problem.

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